Response to Expert Report by Dr. Barber on the North Carolina State Legislature Redistricting Plans

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1 Introduction

The report by Dr. Michael Barber begins with a discussion of the political geography of the state of North Carolina. He emphasizes the heterogeneity of the state. While he points out the strengths of ensemble methods to separate the effect of natural clustering of votes and other effects due to political geography, Dr. Barber limits its use to analysis of the individual county clusters. Similarly, though he uses a collection of election data at the cluster level, he does not consider a diverse collection of election analyses both at the cluster level and when performing his statewide analysis. Rather, he restricts himself to a single summary statistic, namely, counting the number of Democratic-leaning districts at the individual cluster level based primarily on a composite election obtained through averaging several past statewide elections.

We complete the missing parts of Dr. Barber's analysis using data directly from his report when possible. When needed, we augment this data with an ensemble of maps obtained by running Dr. Barber's code. From this completed analysis, we see that Dr. Barber's ensemble shows both the Enacted NC House and the Enacted NC Senate to be extreme partisan outliers with a clear and systematic tilt in favor of electing Republicans.

When we focus on the structure of the enacted maps in the county clusters under Dr. Barber's analysis, we again see the same structures we observed using the Primary Ensembles from our initial report. These structures showed the enacted map to be an extreme outlier. Due to time constraints, we did not complete cluster level analysis on all clusters using Dr. Barber's simulations; we have, however, performed a cluster level analysis on a diverse collection of clusters in the NC House. Our cluster level analysis considers not only seat counts, but also the margins of victory within those seats. By examining the margins, we identify extreme partisan behavior at the cluster level using the very sampling code that Dr. Barber created.

We conclude that Dr. Barber's ensembles provide another independent verification that the enacted plans for the NC House and NC Senate are extreme gerrymanders.

2 Comment on Political Geography of State

In Section 3 of Dr. Barber's report, he discusses the political geography of the state. He made a number of statewide evaluations of the partisan structure using a single average of 11 statewide elections from 2014-2020. As his analysis in

later sections makes clear, the political climate varies significantly from year to year and election to election. The average of these elections creates a new set of voting data, possibly quite district from those averaged to create it. I see no reason to elevate the behavior and properties of a map under the one particular political environment signified by this vote over other elections. It is important that the map used to translate our election votes into elected officials act in a non-biased way across a number of elections which represent different political climates seen in North Carolina, not just one.

In the rest of the report, Dr. Barber does switch to considering a number of distinct elections. However, he does not return to any aggregate statewide discussion using these individual elections and the diversity of election environments they represent. He does firmly endorse the use of a computer drawn ensemble of maps to create a base line against which the enacted map can be compared. He correctly represents that this method has the advantage of taking into account all of the political geography of the state, such as the concentrating of particular voters in some regions of the state or the preservation of counties and the like. Hence, when a map is an outlier compared to a computer drawn ensemble, these natural clustering or political geography considerations cannot be the explanation.

Dr. Barber never conducts any statewide analysis under his ensemble using different election results. However, all of the components necessary to perform such analysis are present in his report. Utilizing Dr. Barber's cluster-by-cluster ensembles, we complete the absent statewide analysis to examine the number of Democratic leaning seats under various elections. This analysis demonstrates that the enacted map *is* an extreme outlier when compared to Dr. Barber's ensemble.

3 Nonpartisan Ensemble Generated by Dr. Barber

In analyzing the North Carolina State House and Senate maps, Dr. Michael Barber generates an ensemble of non-partisan redistricting maps via the Sequential Monte Carlo (SMC) procedure in the *redist* R-package developed and maintained by a research group at Harvard University. When used to sample from a known distribution in a moderate sized problem, this method has been shown to faithfully sample the target distribution. This was validated on moderate sized examples using an enumeration algorithm developed by the same group that developed the *redist* R-package at Harvard. The method we used has similarly been validated using this and other methods. Dr. Barber used the ensemble method only at the cluster level and does not use it to perform a statewide analysis based on a statewide ensemble. Rather he just summarizes the cluster by cluster results in a few tables (Table 2 and Table 32) instead of performing any analysis which would show the cumulative effect at the statewide level. The coin flipping analogy we offer below shows why this is so inadequate. In utilizing Dr. Barber's ensemble, we demonstrate that he would have concluded the enacted map was an extreme outlier at the statewide level. This is not an endorsement of any of the particular algorithm choices he has made, but rather to demonstrate that this conclusion is available from his findings.

By taking the percentages in the cluster-by-cluster tables in Dr. Barber's report, we were able to perform the statewide analysis he neglected using his data. We were also able to perform this for the collection of different statewide elections Dr. Barber used in his analysis. This allowed us to see the behavior of the maps under different types of elections. Both of these considerations are important and we briefly discuss them individually before turning to the statewide analysis using Dr. Barber's data.

- **Importance of statewide analysis:** Dr. Barber analyzes each cluster one-by-one and concludes that the majority of them are not extreme outliers so under his election composite the map is not an outlier. However, in almost every case, he finds that the more Republican of the non-outlying options is selected. Consider the following analogy. Someone flips a coin that they claim is fair but is in fact biased to produce heads more often. They flip the coin and produce 40 heads and zero tails. On each flip, the chance of getting a head from a fair coin is 50%. Hence the outcome on each flip is not that surprising. Dr. Barber's analysis is analogous to looking at each flip alone and then claiming that the coin is fair because the outcome was a head and the chance of a fair coin producing a head was reasonable. However, taking a more global view one can an easily see that the chance of getting 40 heads in a row is astronomically small. And thus, one can conclude the coin is biased. This would even be true if there were only 35 heads and 5 tails.

  Analogously, each cluster taken individually might not be an extreme outlier, but it is extremely unlikely that all of these clusters woud exist together in a statewide map drawn without partisan intent.

  We will also see that some of the local clusters are extreme outliers in their own right using Dr. Barber's data and extending his analysis to look at the margins of victory (or the extent of the partisan lean) rather than only focusing on the number of seats won by either party (or the direction of the partisan lean). This extended analysis agrees with the finding in our initial report.

- **Often extreme behavior is apparent in only some elections:** If one wanted to rig a card game by colluding with some of the other players, the group would only need to act when none of the group was going to win. The group need only act when cards were aligned against them. Hence, the behavior of a gerrymandered map might appear typical in settings where the gerrymandering party is content with the outcome that one would typically expect without gerrymandering. Furthermore, it is possible that whatever system the card players are using is not sufficient to counteract some hands. In other words, even a card player that is cheating might not be able to win when their opponent draws a royal flush. Hence, it is not to be expected that in all cases a gerrymandered map is effective in supporting the gerrymandering party.

  In particular, one can not simply declare that a map is not gerrymandered because it is fair in some fraction (even a relatively large fraction) of the election environments. If it is clearly gerrymandered in some reasonable and pertinent election environments, then the map should be seen as gerrymandered. To do otherwise would be to argue that a casino would be happy with card players who only cheated 30% of the time and in particular did not cheat when they were already winning or had an unsalvageable hand.

In addition to generating a statewide analysis using the actual data from Dr. Barber's report, we also employ ensembles generated from the *redist* code base, set up according to Dr. Barber's analysis scripts.[1] We then show that well-established methods of probing for gerrymandering reveal that many of the individual clusters are indeed extreme gerrymanders. In doing so, we consider the partisan seat counts of each party and also extend the analysis to consider *how* the seats are won. The latter is important as it shows the degree that a given district is politically safe as well as determines how future political swings, unseen at present, might affect political outcomes. For example, atypically polarized districts can lead to maps which do not respond to the shifts in the electorate's preferences, and effectively lock in a particular outcome. Additionally, when a map has an extremely partisan structure, this can speak to the intent of the map makers even if the structure would be unlikely to affect some collection of elections such as wave elections in favor of the gerrymandering party.

[1] Dr. Barber did include a R Data file which might have included the maps he generated in his run. However, since our version of R was slightly different than his, it would not load. Hence we were forced to re-run his code.

4 Statewide Analysis of Dr. Barber's Ensemble of NC House Plans

Within each cluster, Dr. Barber presents the fraction of plans in his ensembles that would lead to a certain number of Democratic districts under each set of historic and averaged vote counts. These tables can be used to construct the probability of drawing a non-partisan plan at the statewide level that would yield a certain number of Democratic leaning districts under various elections.

Beginning with his averaged statewide vote counts, we construct the statewide probabilities of electing various numbers of representatives and present them in Figure 1 in terms of the number of Democrats elected. Only 0.177% of all of the plans in Dr. Barber's ensemble elect the same or more Republicans than the enacted plan.

Note that our count of Democrats elected includes the Democrats elected in single-district clusters, which are omitted from Dr. Barber's Table 2. So our Figure 1 reports that the enacted plan elects 49 Democrats under Dr. Barber's composite of elections, which is the four Democrats elected in single-district clusters that Dr. Barber reports in his Table 1 plus the 45 Democrats elected in multi-district clusters that Dr. Barber reports in his Table 2.

We repeat the above analysis with the 2016 and 2020 election data used by Dr. Barber. The only supplemental data we introduce is the number of single district Democratic clusters in each election which we have taken from our previous analysis. We summarize the 10 elections in Figure 2 and Table 1.

As in our previous analysis, we find that the outlier status of the ensemble has a significant impact on the amount of power the Republicans can amass in the House. For example, under the votes of the 2020 Lt. Governor race, 2016 Presidential race, and 2020 US Senate race, the ensemble breaks a Republican supermajority in 99.3937%, 98.976, and 99.992% of the plans in Dr. Barber's ensemble, respectively. However, the enacted plan would elect a Republican supermajority under each of these votes. Similarly, under the 2020 Governor race, the Republican majority would have been broken in 96.42% of the plans in Dr Barber's ensemble, yet they would have maintained the majority using the enacted map under these votes.

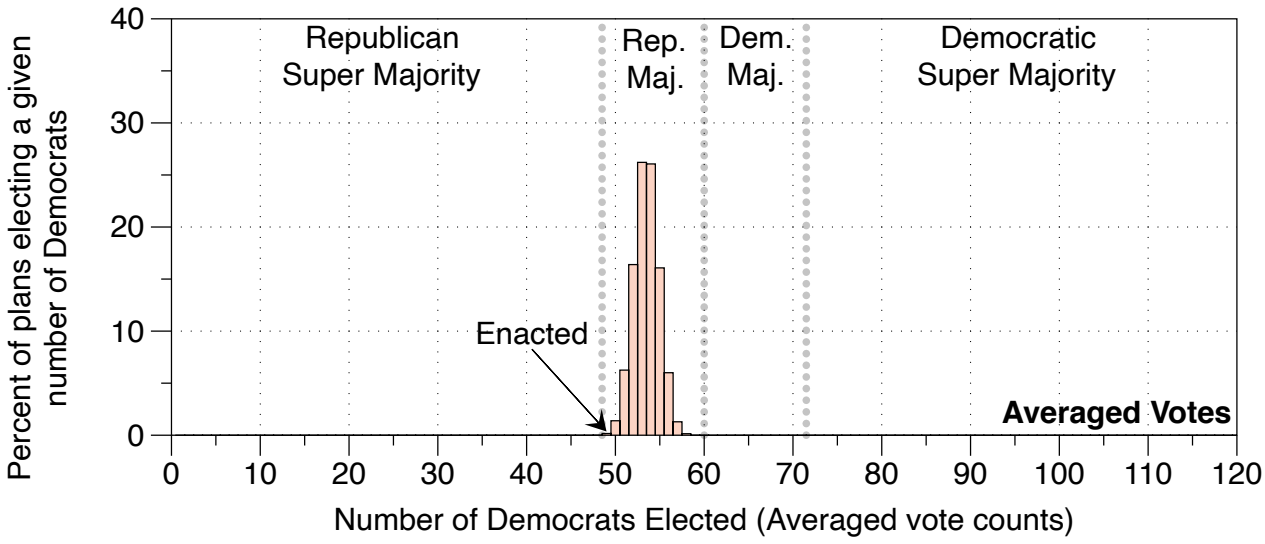


Figure 1: We compare Dr. Barber's statewide ensemble with the enacted plan under the Averaged election results used in his report. We find that only 0.177% of all of the plans in his ensemble would elect the same or more Republicans.

Election	Statewide Dem. Vote	% of Dr. Barber's Plans electing the same or more Republicans than the enacted plan
Barber's Average Vote	-	0.177%
2020 Governor	52.32%	0.204%
2016 Attorney General	50.20%	1.34%
2020 Attorney General	50.13%	0.00684%
2016 Governor	50.047%	0.215%
2020 President	49.36%	0.000146%
2020 Senate	49.14%	0.00804%
2020 Lt. Governor	48.40%	0.000377%
2016 President	48.024%	1.02%
2016 Senate	46.98%	0.223%
2016 Lt. Governor	46.59%	0.518%

Table 1: When considered at the statewide level, the ensembles produced by Dr. Barber are all extreme outliers. The chance that a plan drawn from the ensemble would elect the same or more Republicans as the enacted plan is, at most, 1.34%; in all but three of the elections it is less than 0.25%. We have ordered the elections with the election with the largest Democratic statewide vote fraction at the top and the election with largest Republican statewide vote fraction at the bottom. It is worth noting that many of the most extreme outliers happen for those between 50% and 48%. Looking at Figure 2, we see that this is the range where the Republicans would typically lose the super majority according to Dr. Barber's analysis. Though "Barber's Average Vote" which he used as a partisan index might or might not represent an actual plausible voting pattern, we have included it for comparison.

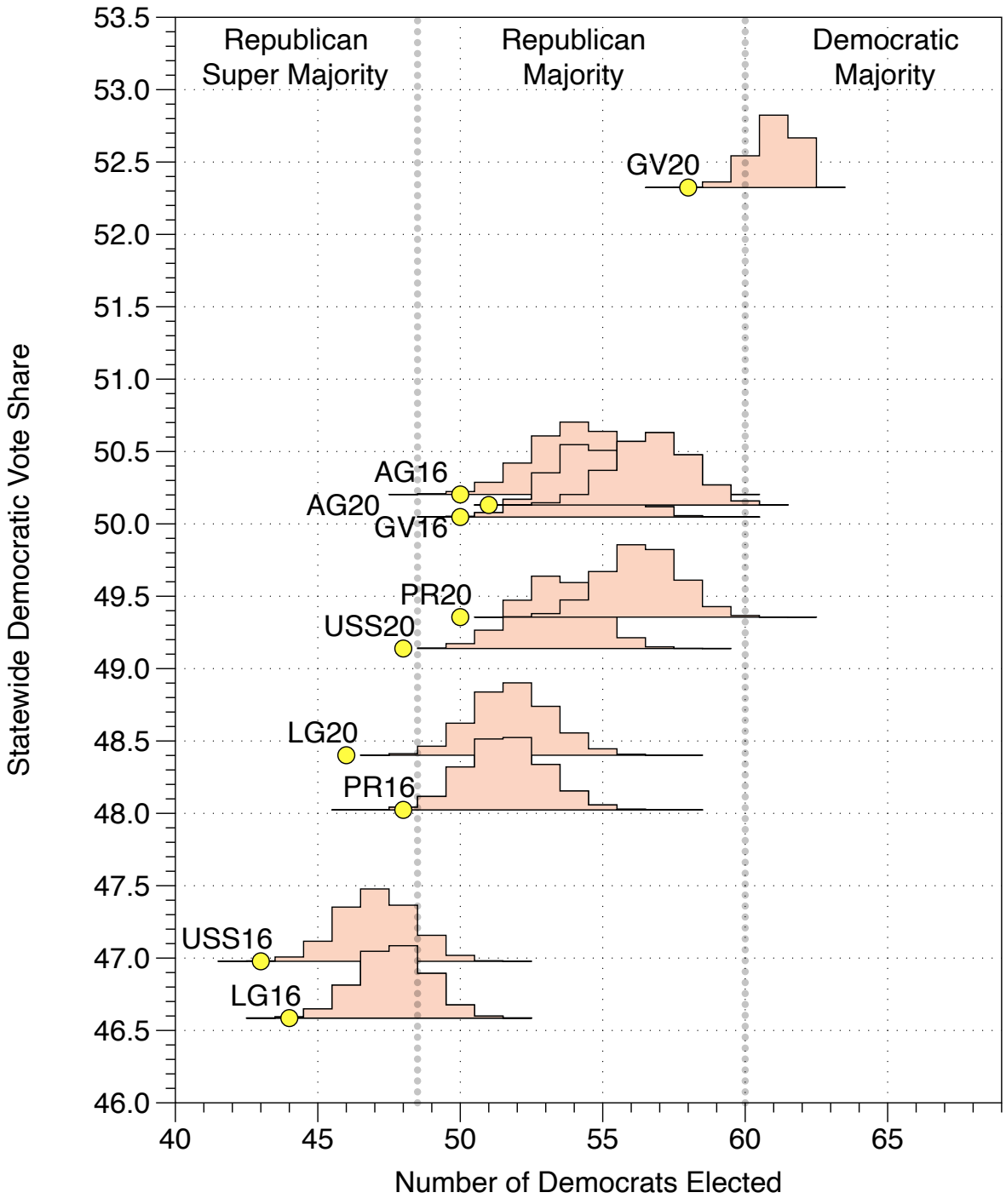


Figure 2: We compare Dr. Barber's statewide ensemble with the enacted plan under the ten 2016 and 2020 elections used in his report. Yellow dots show the result of the enacted plan. The enacted plan is an extreme outlier when considering the same data under a statewide lens. We summarize the numerical extent of the outliers in Table 1. The elections are abbreviated with the last two digits signifying the year, and the first letters representing Lt. Governor (LG), Governor (GV), President (PR), and US Senate (USS).

5 Statewide Analysis of Dr. Barber's Ensemble of NC Senate Plans

Repeating the above analysis for Dr. Barber's ensemble of Senate plans, we begin with the averaged statewide vote counts. We construct the statewide probabilities of electing various numbers of Senators and present them in Figure 3. Once again, our count of Democrats elected includes the Democrats elected in single-district Senate clusters, which are omitted from Dr. Barbers Table 32. So our Figure 3 reports that the enacted plan elects 20 Democrats under Dr. Barbers composite of elections, which is the four Democrats elected in single-district clusters that Dr. Barber reports in his Table 31 plus the 16 Democrats elected in multi-district clusters that Dr. Barber reports in his Table 32. Only 0.00385% of all of the plans in Dr. Barber's ensemble elect the same or more Republicans. Furthermore, this is the percentage of plans that lead to a Republican supermajority under these votes (which the enacted plan would produce as well). In other words, while the enacted plan always produces a Republican supermajority under Dr. Barber's analysis, only .00385% of the non-partisan plans that Dr. Barber simulates would produce a Republican supermajority.

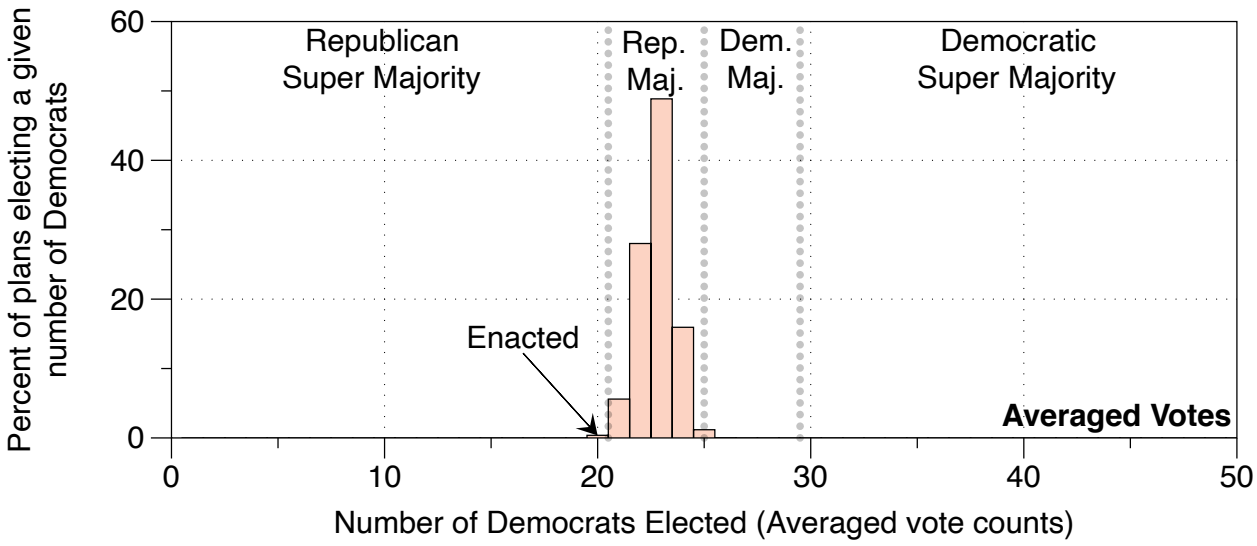


Figure 3: We compare Dr. Barber's statewide ensemble with the enacted plan under the Averaged election results used in his report. We find that only 0.00385% of all of the plans in his ensemble would elect the same or more Republicans than the enacted plan.

We repeat the above analysis with the 2016 and 2020 election data used by Dr. Barber. The only supplemental data we introduce is the number of single district Democratic clusters in each election which we have taken from our previous analysis. We summarize the 10 elections in Figure 4 and Table 2.

Again, we find that the outlier status of the ensemble has a significant impact on the amount of power the Republicans can amass in the Senate. Under the votes of the 2016 Governor race and 2016 Attorney General races, the Republicans lose their supermajority in 99.9544% and 98.9501% of the plans in Dr. Barber's ensemble, respectively. However, the enacted plan would elect a Republican supermajority under each of these voting patterns.

Election	Statewide Dem. Vote	% of Dr. Barber's Plans electing the same or more Republicans than the enacted plan
Averaged	-	0.00385%
2020 Governor	52.32%	1.92%
2016 Attorney General	50.20%	1.05%
2016 Governor	50.047%	0.047%
2020 Attorney General	50.13%	3.74%
2020 President	49.36%	9.92%
2020 Senate	49.14%	5.76%
2020 Lt. Governor	48.40%	0.250%
2016 President	48.024%	0.16%
2016 Senate	46.98%	1.22%
2016 Lt. Governor	46.59%	10.9%

Table 2: When considered at the statewide level, many of the ensembles produced by Dr. Barber are extreme outliers. In six of the ten elections, there is less than a 2% chance that a plan drawn from the ensemble would elect the same or more Republicans as the enacted plan; in three of the ten elections, there is less than a 0.251% chance that a plan drawn from the ensemble would elect the same or more Republicans than the enacted plan. As we have remarked in both our original report and in the analysis below, this *does not* mean that the enacted plan is not an extreme partisan gerrymander under the other four elections; it only indicates that the plan is not as extreme of an outlier in these elections under the particular lens of seat counts.

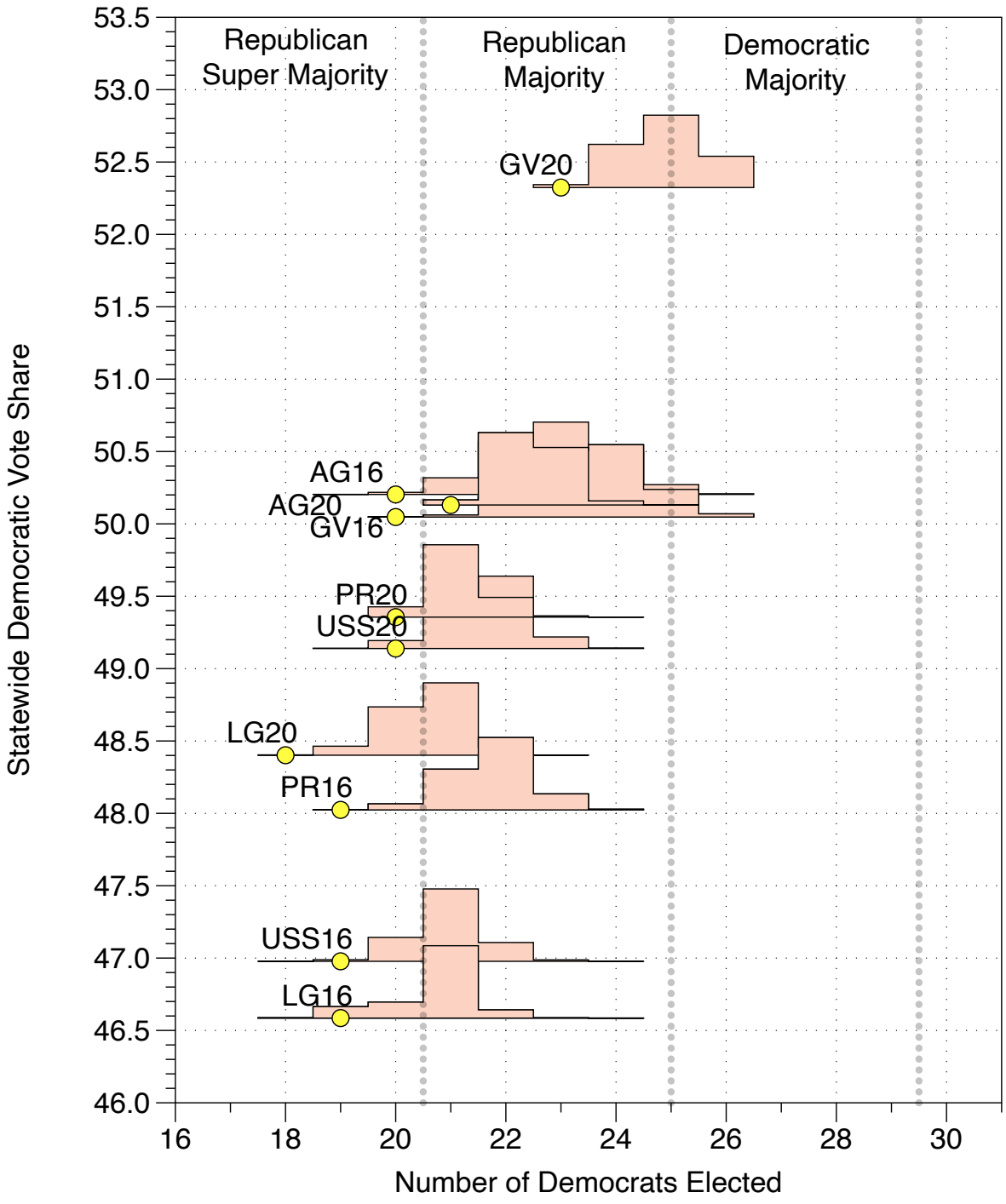


Figure 4: We compare Dr. Barber's statewide ensemble with the enacted plan under the ten 2016 and 2020 elections used in his report. Yellow dots show the result of the enacted plan. The enacted plan is an extreme outlier when considering the same data under a statewide lens. We summarize the numerical extent of the outliers in Table 1. The elections are abbreviated with the last two digits signifying the year, and the first letters representing Lt. Governor (LG), Governor (GV), President (PR), and US Senate (USS).

6 Cluster by Cluster Analysis

We now turn to examining certain clusters presented in Dr. Barber's work. We do not exhaustively examine all of the clusters. Rather, we select certain clusters to demonstrate how the lens that Dr. Barber chooses to use (namely only looking at the number of Democratic districts) yields an incomplete picture of the partisan make up of the districts even with respect to the individual districts.

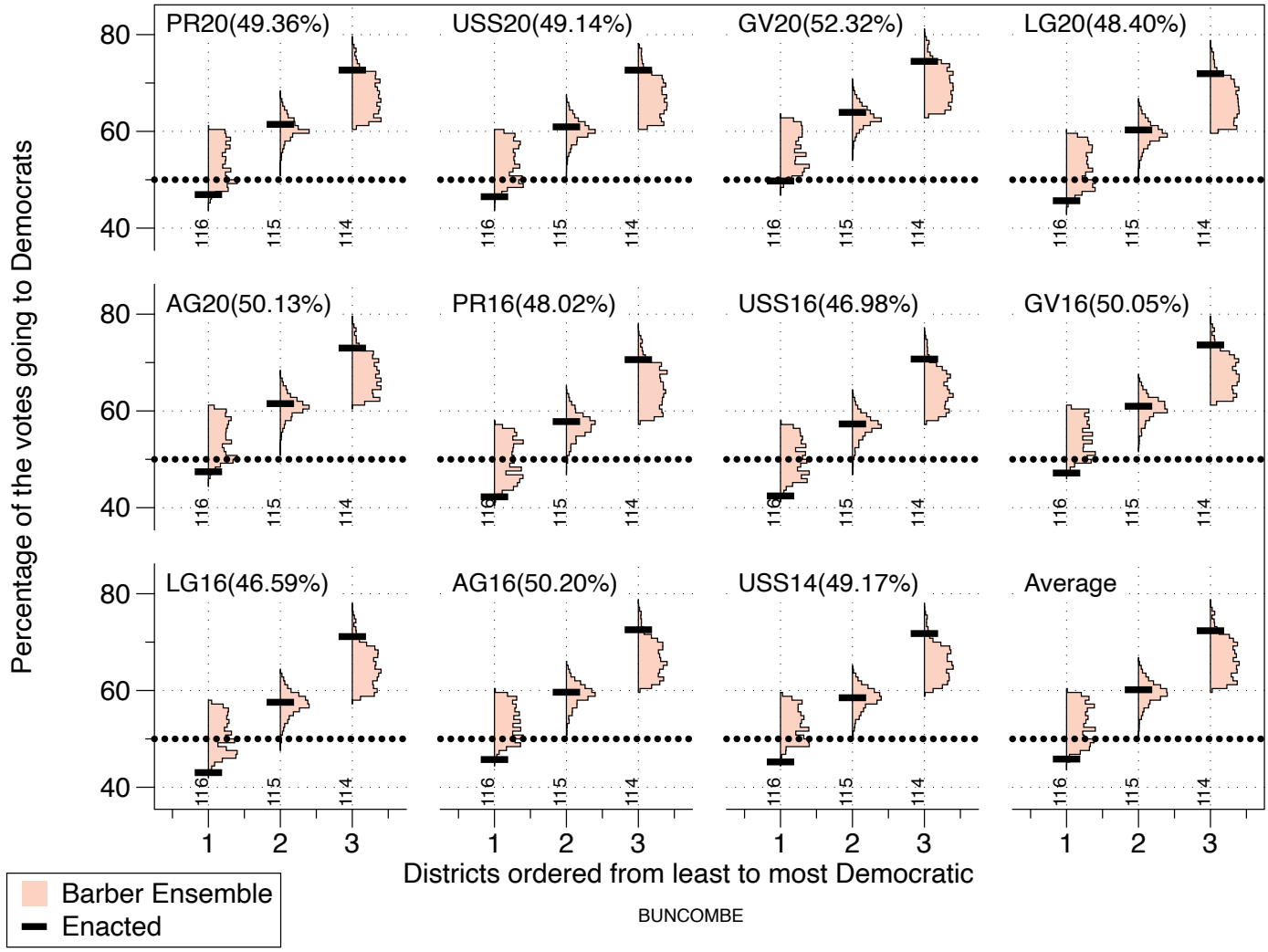
For a more complete picture, one would need to look at the actual partisan make-up of each district within a cluster. In fact, Dr. Barber reported on these values for the enacted plan, but did not compare these values to those found in his ensemble. One way of comparing these numbers is to examine the rank ordered marginal distributions of the vote fraction in each district. To do this, we order the districts from least to most Democratic (what Dr. Barber calls the Partisan Lean of Districts), and then look at the distribution of the most Republican, second most Republican, etc..., all the way until we reach the most Democratic district.

This type of analysis reveals not only how many Democratic leaning districts are within Dr. Barber's ensemble, but also *how much* they lean Democratic (or Republican). As we have demonstrated in our report, this is also relevant at a statewide level.

Note that all of our previous statewide analysis of seat counts simply relied on the numbers presented in Dr. Barber's report, i.e., the exact same ensemble that he relies on. The analysis below uses an ensemble of plans derived from running Dr. Barbers code (we were unable to extract his ensembles he used from the data he provided).[2] However, re-running his same code with his exact same input parameters should produce a comparable ensemble to the one he generated from the report, assuming that his code performs in the way he represents.

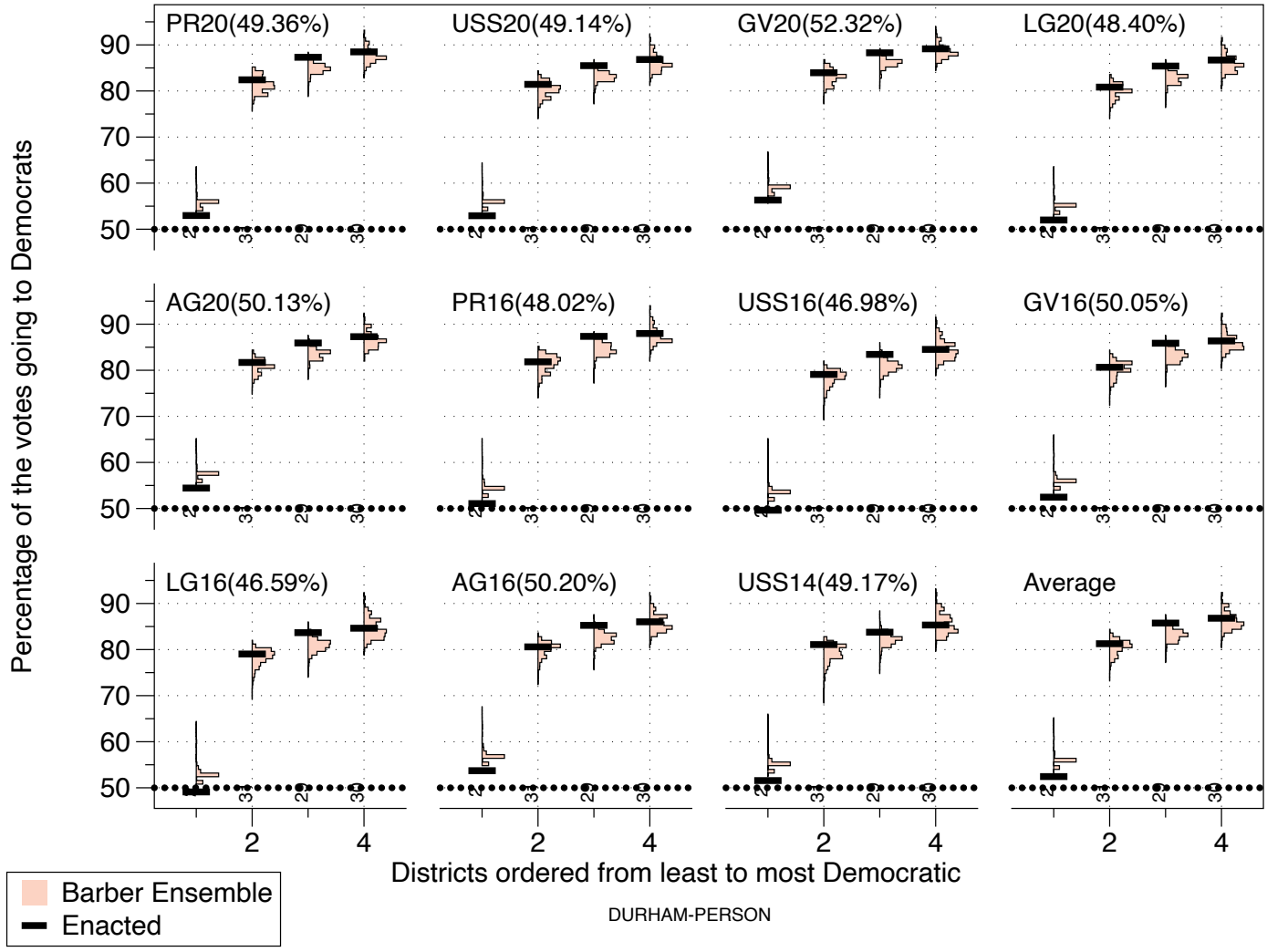
The main conclusion is that when comparing the cluster-by-cluster results from Dr. Barber's ensemble to those in our report, we find the qualitative structure to be the same. We again conclude that the enacted map is an extreme outlier when using Dr. Barber's ensemble with this additional analysis. We include a number of county clusters from the NC House. We make a number of comments in the caption of each figure. We refer the reader to our initial report to the court for a description of these Ranked-Ordered-Marginal-Histograms.

[2]We obtained the ensemble data from runs of Dr. Barber's code from Wes Pegden (CMU) who ran the code on his R installation as we did not have a computing environment able to run the code conveniently during the window when the rebuttal reports were due.



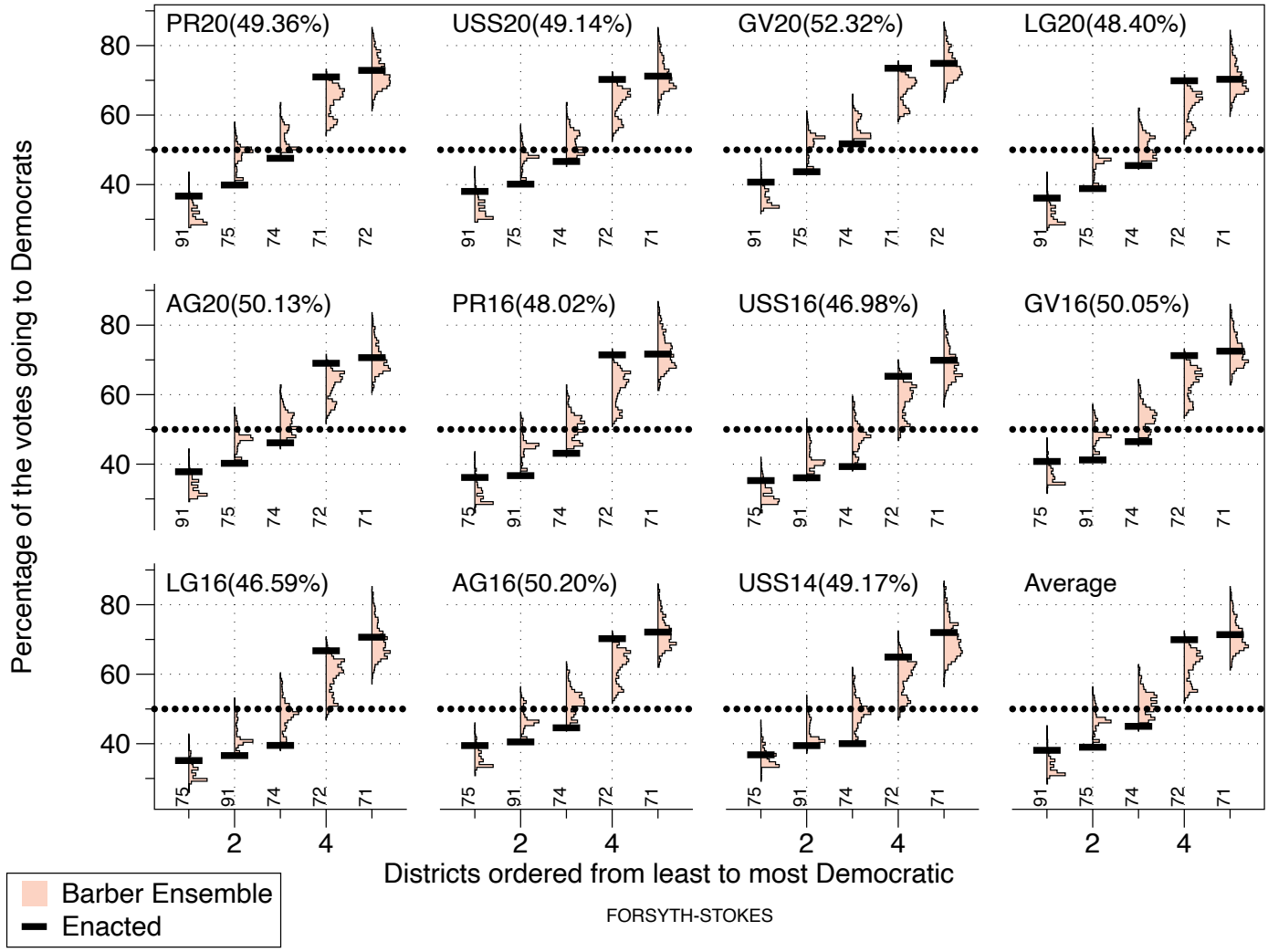
Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	107	0.277	2409	6.23	38664	1	3
PR20	756	1.96	3095	8.0	38664	1	3
USS20	409	1.06	2529	6.54	38664	1	3
GV20	662	1.71	3200	8.28	38664	1	3
LG20	424	1.1	2624	6.79	38664	1	3
AG20	534	1.38	2655	6.87	38664	1	3
PR16	321	0.83	2701	6.99	38664	1	3
USS16	17	0.044	2062	5.33	38664	1	3
GV16	18	0.0466	2067	5.35	38664	1	3
LG16	18	0.0466	1998	5.17	38664	1	3
AG16	17	0.044	1992	5.15	38664	1	3
USS14	3	0.00776	1807	4.67	38664	1	3

Figure 5: In Buncombe County, the Enacted maps is an extreme outlier under Dr. Barber's ensemble. We see the same structure as we saw when compared with the probability ensemble our initial report. The most Republican district in the enacted plan has exceptionally few Democrats while the most Democratic district has exceptionally many Democrats. The result is that the Democrats never win three seats in the enacted plan under any of the elections considered, including Dr. Barber's composite "Averaged Election", even though they would typically do so under a number of elections under Dr. Barber's ensemble.



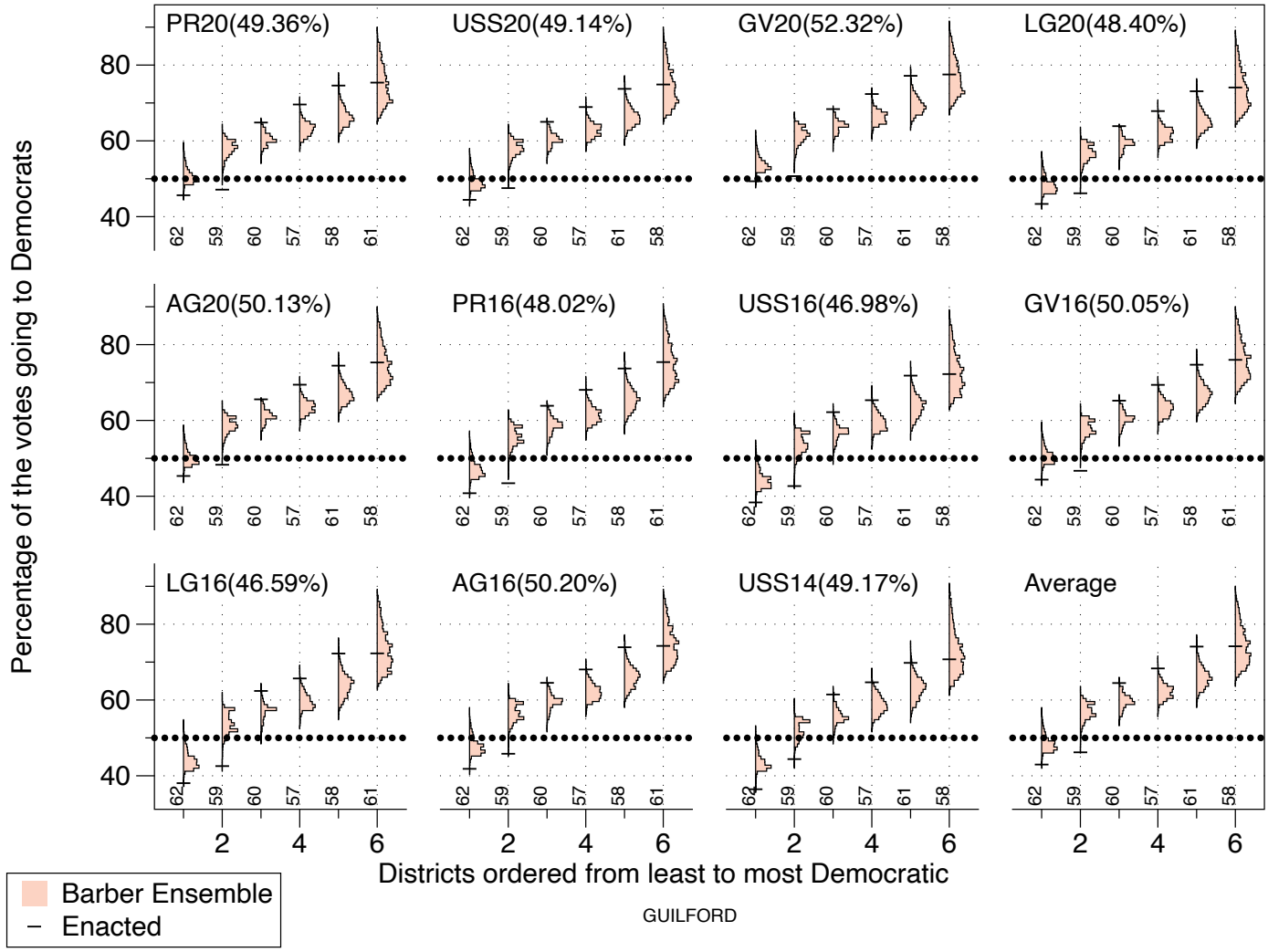
Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	0	0.0	1396	3.69	37800	1	3 4
PR20	0	0.0	790	2.09	37800	1	3 4
USS20	0	0.0	1326	3.51	37800	1	3 4
GV20	0	0.0	1123	2.97	37800	1	3 4
LG20	0	0.0	1199	3.17	37800	1	3 4
AG20	0	0.0	1205	3.19	37800	1	3 4
PR16	0	0.0	1184	3.13	37800	1	3 4
USS16	0	0.0	2932	7.76	37800	1	3 4
GV16	0	0.0	1382	3.66	37800	1	3 4
LG16	0	0.0	2675	7.08	37800	1	3 4
AG16	0	0.0	1931	5.11	37800	1	3 4
USS14	0	0.0	10357	27.4	37800	1	3 4

Figure 6: In the Durham-Person cluster, we the same outlier structure in the enacted map when compared to Dr. Barber's ensemble as when compared to the primary ensemble in our orignal report. We see that the most Republican district has been depleted of Democrates. This makes the district much more competitive than it typically would be under a non-partisan redistricting plan.



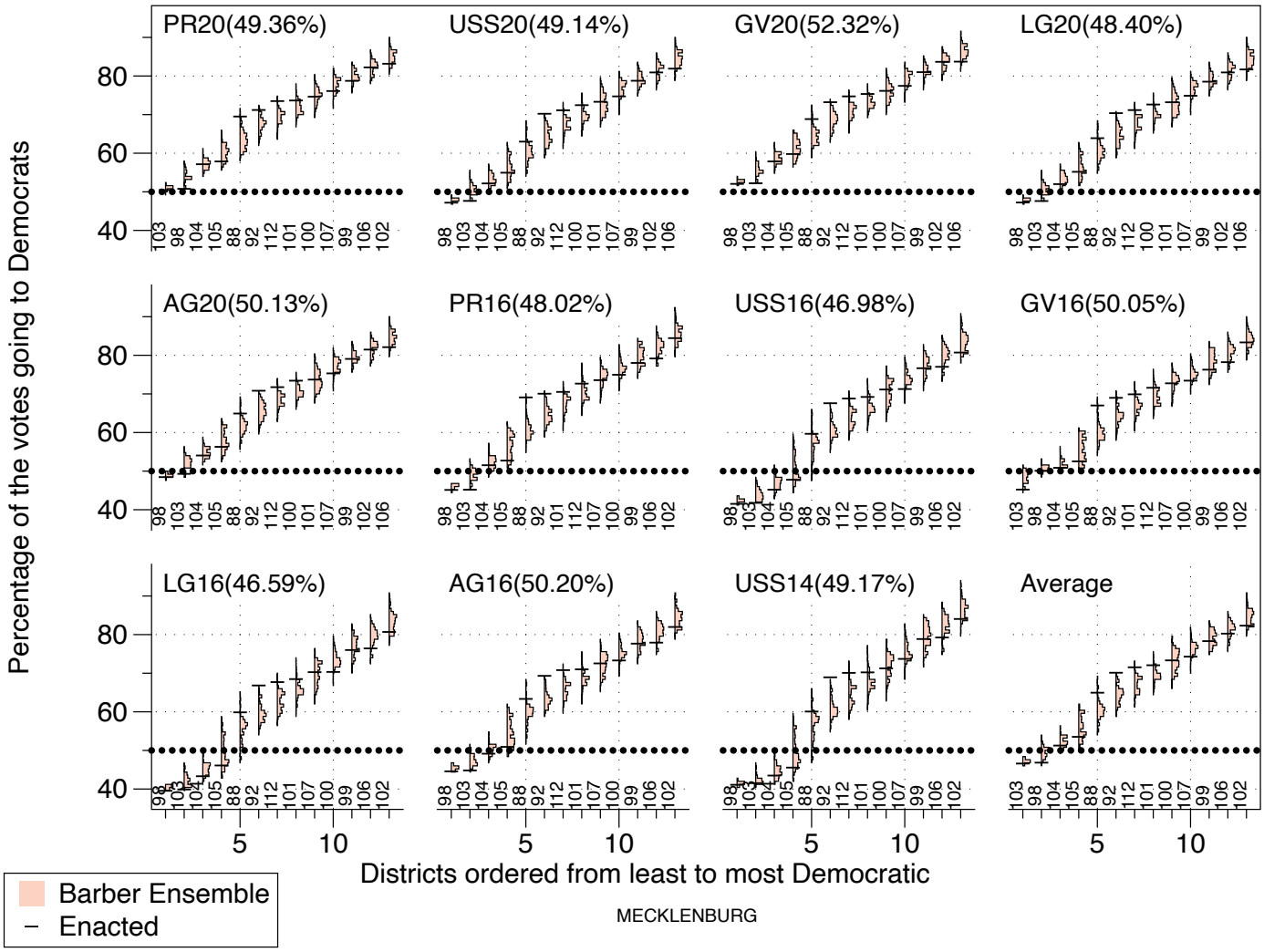
Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	17	0.456	317	8.51	3726	1 2 3	4 5
PR20	4	0.107	349	9.37	3726	1 2 3	4 5
USS20	60	1.61	429	11.5	3726	1 2 3	4 5
GV20	2	0.0537	357	9.58	3726	1 2 3	4 5
LG20	21	0.564	376	10.1	3726	1 2 3	4 5
AG20	47	1.26	395	10.6	3726	1 2 3	4 5
PR16	7	0.188	284	7.62	3726	1 2 3	4 5
USS16	44	1.18	280	7.51	3726	1 2 3	4 5
GV16	11	0.295	292	7.84	3726	1 2 3	4 5
LG16	30	0.805	269	7.22	3726	1 2 3	4 5
AG16	25	0.671	263	7.06	3726	1 2 3	4 5
USS14	13	0.349	351	9.42	3726	1 2 3	4 5

Figure 7: In the Forsyth-Stokes cluster, We again see the same structure in Dr. Barber's ensemble as in the primary ensemble from our initial report. We see abnormally few Democrats in the second and third most Republican districts while we see abnormally many Democrats in the most Republican district and in the two most Democratic districts. The effect is to regularly flip the 3rd most Republican district to the republicans under the enacted map even under elections where many to almost all of the plans in Dr. Barber's ensemble would have awarded the seat to the Democrats.



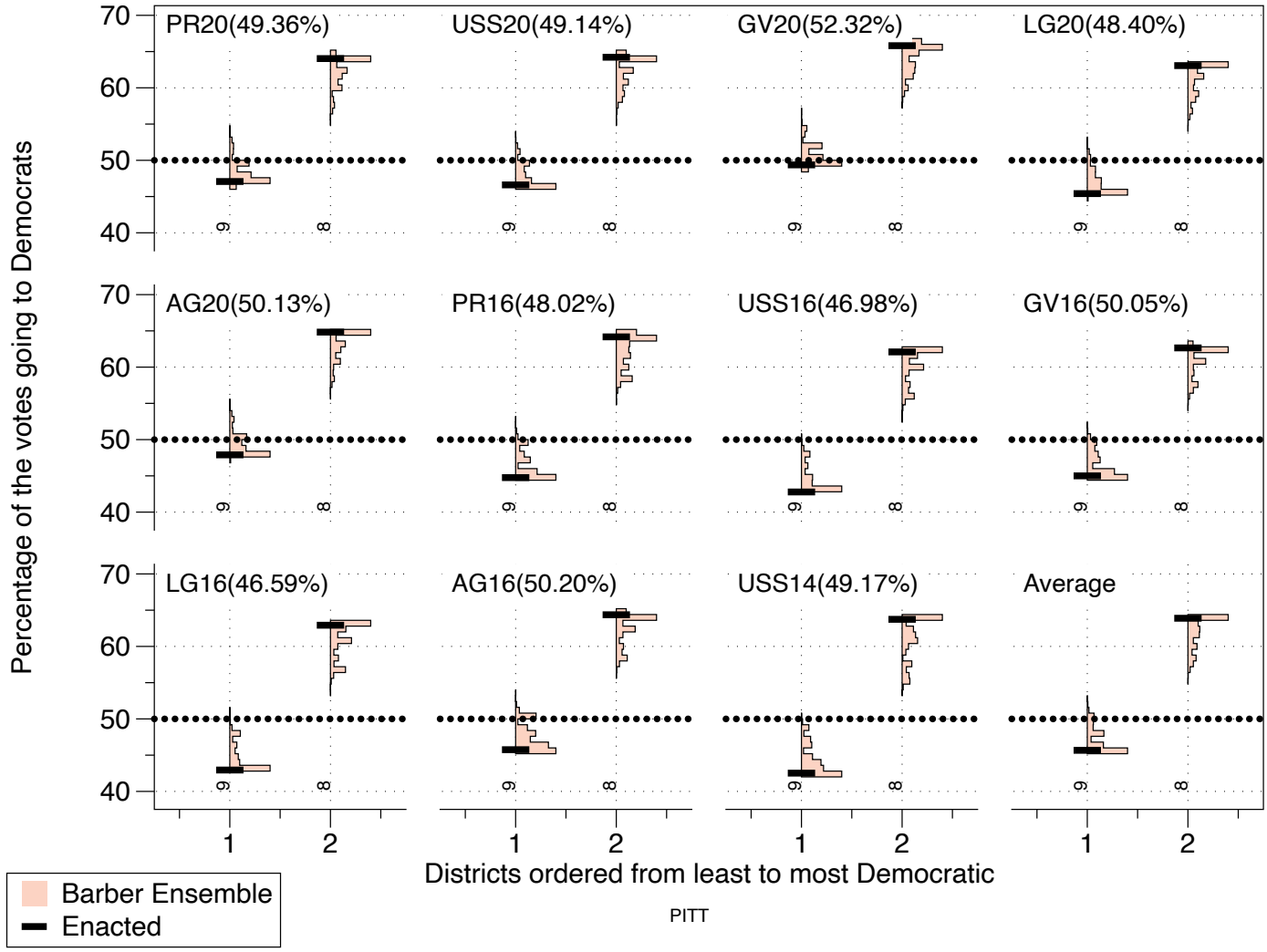
Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	0	0.0	0	0.0	15489	1 2	3 4 5 6
PR20	0	0.0	0	0.0	15489	1 2	3 4 5 6
USS20	0	0.0	0	0.0	15489	1 2	3 4 5 6
GV20	0	0.0	0	0.0	15489	1 2	3 4 5 6
LG20	0	0.0	0	0.0	15489	1 2	3 4 5 6
AG20	0	0.0	0	0.0	15489	1 2	3 4 5 6
PR16	0	0.0	0	0.0	15489	1 2	3 4 5 6
USS16	0	0.0	0	0.0	15489	1 2	3 4 5 6
GV16	0	0.0	0	0.0	15489	1 2	3 4 5 6
LG16	0	0.0	0	0.0	15489	1 2	3 4 5 6
AG16	0	0.0	0	0.0	15489	1 2	3 4 5 6
USS14	0	0.0	0	0.0	15489	1 2	3 4 5 6

Figure 8: Dr. Barber did identify Guilford county as a Republican Gerrymander in the enacted map. The structure which produces this result is clear when compared with this plot of Dr. Barber's ensemble. We see that the two most Republican districts have abnormally few Democrats and the next three Republican districts have abnormally many Democrats. The effect is that the second most Republican seat reliably goes to the Republican party even though in some elections almost all of the maps in Dr. Barber's ensemble would award the seat to the Democrats. This was the same structure seen in the plots of our primary ensemble from our initial report.



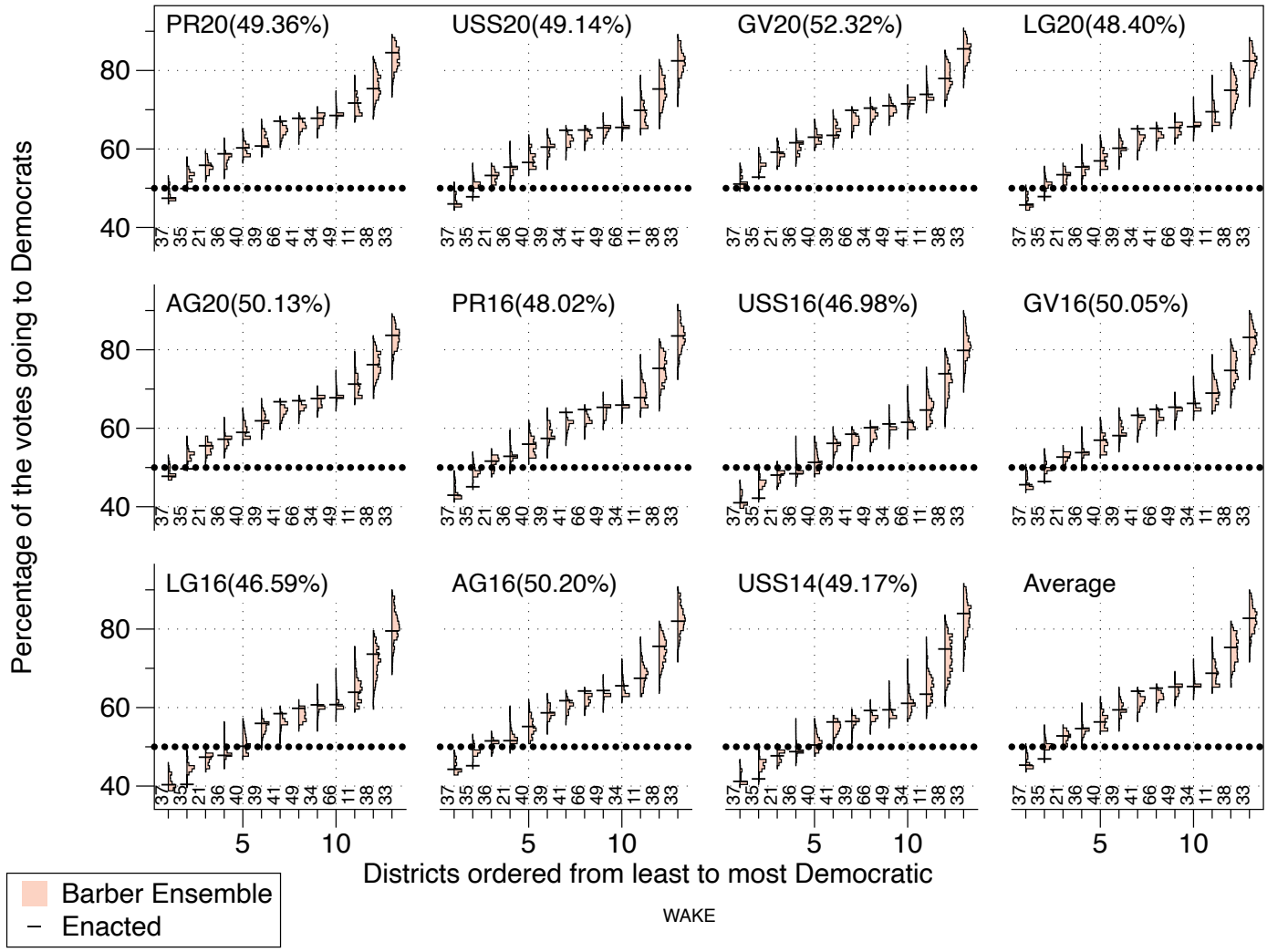
Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	139	4.4	14	0.443	3161	1 2 3 4	5 6 7 8
PR20	105	3.32	18	0.569	3161	1 2 3 4	5 6 7 8
USS20	145	4.59	29	0.917	3161	1 2 3 4	5 6 7 8
GV20	114	3.61	17	0.538	3161	1 2 3 4	5 6 7 8
LG20	117	3.7	17	0.538	3161	1 2 3 4	5 6 7 8
AG20	119	3.76	17	0.538	3161	1 2 3 4	5 6 7 8
PR16	23	0.728	18	0.569	3161	1 2 3 4	5 6 7 8
USS16	74	2.34	15	0.475	3161	1 2 3 4	5 6 7 8
GV16	56	1.77	23	0.728	3161	1 2 3 4	5 6 7 8
LG16	68	2.15	18	0.569	3161	1 2 3 4	5 6 7 8
AG16	52	1.65	15	0.475	3161	1 2 3 4	5 6 7 8
USS14	153	4.84	16	0.506	3161	1 2 3 4	5 6 7 8

Figure 9: In Mecklenburg county, we again have that the four most Republican districts have abnormally few Democrats in them while the next four most Republican districts have abnormally many Democrats. This is the same structure as we saw under our primary ensemble in our initial report. The effect is that in a number of elections the Republican party wins one to two more seats than the typical plan from Dr. Barber's ensemble would award.



Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	314	6.05	1929	37.2	5189	1	2
PR20	1539	29.7	1974	38.0	5189	1	2
USS20	1525	29.4	1929	37.2	5189	1	2
GV20	1556	30.0	1974	38.0	5189	1	2
LG20	1537	29.6	1974	38.0	5189	1	2
AG20	1537	29.6	1974	38.0	5189	1	2
PR16	483	9.31	1929	37.2	5189	1	2
USS16	0	0.0	1660	32.0	5189	1	2
GV16	483	9.31	1929	37.2	5189	1	2
LG16	0	0.0	1660	32.0	5189	1	2
AG16	169	3.26	1660	32.0	5189	1	2
USS14	0	0.0	1660	32.0	5189	1	2

Figure 10: In Pitt county we see that same structure we found in our Primary ensemble repeated in Dr. Barber's ensemble. In particular, we see the districts pulled to the extremes of what is seen in Dr. Barber's ensemble. The depletion of Democrats from the more Republican district protects it from electing a Democrat in the enacted plan even though it would elect a Democrat in many of the plans in Dr. Barber's ensemble in a few of the elections we considered.



Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	159	1.11	2649	18.5	14305	1 2	3 4 5 6 7 8
PR20	140	0.979	1872	13.1	14305	1 2	3 4 5 6 7 8
USS20	209	1.46	2961	20.7	14305	1 2	3 4 5 6 7 8
GV20	145	1.01	1772	12.4	14305	1 2	3 4 5 6 7 8
LG20	159	1.11	2240	15.7	14305	1 2	3 4 5 6 7 8
AG20	165	1.15	2260	15.8	14305	1 2	3 4 5 6 7 8
PR16	137	0.958	2264	15.8	14305	1 2	3 4 5 6 7 8
USS16	196	1.37	3774	26.4	14305	1 2	3 4 5 6 7 8
GV16	220	1.54	3504	24.5	14305	1 2	3 4 5 6 7 8
LG16	196	1.37	2707	18.9	14305	1 2	3 4 5 6 7 8
AG16	205	1.43	3076	21.5	14305	1 2	3 4 5 6 7 8
USS14	287	2.01	3632	25.4	14305	1 2	3 4 5 6 7 8

Figure 11: In Wake county, we see that the number of Democrats in the first two districts is exceptionally low. Looking across the different Ranked Ordered Marginal Histograms, we see that this increases the electoral environments (as captured in different elections) in which the Republican party wins one of these two districts. In particular, Dr. Barber's ensemble would lead to the Democrats typically winning one of these two districts in cases where the enacted plan does not.

7 Comments on Sampling Methods

We now give some additional details to clarify some of the terms we used and the procedures we followed in sampling of the legislative maps in our original report in light of the discussion in Dr. Barber's report.

We recall that in the Legislative case we used parallel tempering to interpolate between a base measure equal to the uniform measure on spanning forests given the county and population constraints and a measure centered on the districts with a compactness similar to the enacted plan. The Primary ensemble for the legislative ensemble reported in the report was the latter of these two ensembles. The first of these ensembles would be the target distribution of the SMC algorithms from the *rdist* package when it is properly configured with resampling included. We took 4 million steps (proposals the Metropolis-Hastings algorithm) at the spanning tree level and 2 million steps on the other levels. We output maps every 25 steps for a total of 160,000 maps in the 4 million step case and 80,000 map in the 2 million step cases. We interpolated between the different ensembles using between 60 and 100 parallel tempering levels. We proposed switching between the parallel tempering levels every 100 steps. In some cases, we ran a number of clusters together in one sampling run and sometimes we ran them separately or is smaller subgroups in a single run. Generally we ran the larger, more compacted clusters such as Wake or Mecklenburg, in this way.[3] As described in the original report, *independent sample reservoirs* were used to split the 60 to 100 levels into computationally feasible chunks. This also improved the mixing and decorrelation properties of our algorithm. The congressional ensemble was drawn from a measure with a compactness weight against the same tree measure that the resampled rdist algorithm would sample. We used 12 parallel temping levels to move between the distribution without a compactness measure and the finial target distribution with the sampling weight. The number of steps was as specified above. The weights and other parameters used in the different run are specified in the header files of the datasets.

[3] For one run in the Senate, we only ran Granville-Wake for 1 million steps as we had strong evidence that this was sufficient for the parameter values being considered.

I declare under penalty of perjury under the laws of the state of North Carolina that the foregoing is true and correct to the best of my knowledge.

Jonathan Mattingly, 12/28/2021